Tutorial 5 (IndefiniteIntegrals+IntegrationTechniques)

(You should practice writing proper steps.)

1.(a) If you know how to differentiate

$$e^{kx},\ \sin kx,\ \cos kx,\ \tan kx,\ \ln(kx+c)$$

then you should be able to carry out the following integrations by observation (i.e., you may be able to write down the final answers fast):

$$\int e^{kx}\,dx,\ \int \sin kx\ dx,\ \int \cos kx\ dx,\ \int \sec^2 kx\ dx,\ \int \frac{1}{kx+c}dx.$$

(b) Practice writing proper steps for integration by using the substitution $u = ax+b$:

$$\int e^{ax+b}\,dx,\ \int \sin(ax+b)\ dx,\ \int \cos(ax+b)\ dx,\ \int \sec^2(ax+b)\ dx,\ \int \frac{1}{ax+b}dx.$$

2. **Integrate.**

(a) $\int (5x^4 - 3x^{-4} + 9x)dx$ (b) $\int (x-1)(x+2)dx$ (c) $\int \left(x+\frac{1}{x}\right)^2 dx$

(d) $\int \left(\frac{2}{x} - 4e^x + \sin 2x\right)dx$ (e) $\int \frac{x^2-3x}{\sqrt{x}}dx$ (f) $\int \frac{e^{3x}-e^x}{e^{2x}}dx$

(g) $\int \left(\sec^2 3x + e^{-2x} + \frac{2}{3x+1}\right)dx$ (h) $\int \frac{3x}{x+2}dx$ [Try writing $3x$ as $3(x + 2)$ + ___.]

3. Integration by substitution.

(You need to decide on a suitable substitution. Write proper steps.)

(a) $\int (4x-5)^{-4}\,dx$ (b) $\int \frac{1}{3x+7}dx$ (c) $\int \frac{10x^4}{\sqrt{2x^5+9}}dx$

(d) $\int \sin(2x+3)dx$ (e) $\int \frac{dx}{x\sqrt{\ln x}}$ (f) $\int \sqrt{2x+1}\,dx$

(g) $\int x\sqrt{x+4}dx$ (h) $\int \frac{(1+\sqrt{x})^9}{\sqrt{x}}dx$ (i) $\int x^2[\cos(x^3+1)]dx$

(j) $\int \frac{x+2}{x^2+4x+8}dx$ (k) $\int x^2\sqrt{x^3+2}\ dx$ (l) $\int \frac{x+3}{x^2+6x+8}dx$

(m) $\int \frac{8x}{(x^2-3)^{\frac{3}{2}}}dx$ (n) $\int x^3(7+x^2)^{5/2}dx$

4. (a) $\int \frac{dx}{x-1} =$ (b) $\int \frac{dx}{x+3} =$ (c) Factor $x^2 + 2x - 3$.

(d) Express $\frac{x}{x^2+2x-3}$ as the sum of its **partial fractions**.

This means writing

$$\frac{x}{x^2+2x-3} \text{ in the form } \frac{A}{x+3} + \frac{B}{x-1}.$$

(e) Do the same for $\frac{5x+5}{x^2+2x-3}$.

(f) Hence, evaluate $\int \frac{x\,dx}{x^2+2x-3}$ and $\int \frac{5x+5}{x^2+2x-3}dx$

5. Use **partial fractions** to assist you in evaluating

(a) $\int \frac{x+4}{x^2-3x+2}dx$ (b) $\int \frac{7x-5}{2x^2-3x+1}dx$ (c) $\int \frac{1}{(x-a)(x-b)}dx$

6. The following requires knowledge of **trigonometric identities**. You will be guided.

(a) $\int \cos^2 x\ dx$ [Use: $\cos 2x \equiv 2\cos^2 x - 1$]

(b) $\int \sin 3x \cos x\ dx$ [Use: $\sin A \cos B \equiv \frac{1}{2}\sin(A-B) + \frac{1}{2}\sin(A+B)$]

(c) (i) What is **Euler's formula**?

(ii) Use $\cos\theta = \frac{e^{i\theta} + e^{-i\theta}}{2}$ to derive the identity: $\cos^3\theta = A\cos 3\theta + B\cos\theta$ for some values of *A* and *B*. Find the values of *A* and *B*.

[$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$ may be useful.]

(iii) Evaluate $\int \cos^3 x\ dx$.

(d) (i) What is **Euler's formula**?

(ii) Use $\cos\theta = \frac{e^{i\theta} + e^{-i\theta}}{2}$ and $\sin\theta = \frac{e^{i\theta} - e^{-i\theta}}{2i}$ to derive the identity:

$\sin 5x \cos 3x = A\sin 8x + B\sin 2x$ for some values of *A* and *B*.

Find the values of *A* and *B*.

(iii) Evaluate $\int \sin 5x \cos 3x\ dx$.

7. Trigonometric functions involved. [*Choose an appropriate substitution*.]

(a) $\int \sin x \cos(\cos x)dx$ (b) $\int x^2 \cos(x^3 + e^2)dx$ (c) $\int \frac{\sin x}{\sqrt[3]{\cos x}}dx$

(d) $\int \tan x \sec^4 x dx$ [Let $u = \tan x$ and $\sec^2 x = 1 + \tan^2 x$ may be useful.]

(e) $\int \cos x \sin^3 x dx$ [Note that $\cos x \sin^3 x = \cos x (\sin x)^3$.]

8. **Integration by Parts**.

(a) $\int xe^x dx$ (b) $\int x \ln x \, dx$ (c) $\int x \cos x \, dx$

(d) $\int x^2 \cos x dx$ [You may need to carry out integration by parts more than once.]

(e) $\int x^3 \ln x dx$

[Some people may need to use integration by parts more than once; see if you can solve this by using it once only.]

(f) $\int e^x \sin x \, dx$

[You may not be able to get the answer directly through integration by parts. Apply integration by parts twice and you may see what I mean; perhaps you would then know how to proceed.]

(g) $\int (x-2)e^{2x+3}dx$

(nby, Nov 2015)